BCG-003-001543

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Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

August – 2021

S - 502 : Statistics

(Mathematical Statistics)

(Old Course)

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Scientific calculator is allowed.
(2) Statistical table will be provide by Institute.

1 Filling the blanks and short questions. (Each 1 mark) **(20)**

1. ___________is a characteristic function of Poisson distribution.
2. ___________is a characteristic function of Standard Normal distribution.
3. ________is a characteristic function of Geometric distribution.
4. __________is a characteristic function of Chi-square distribution.
5. __________is a moment generating function of $\gamma(\alpha, p)$.
6. ____________is a moment generating function of Chi-square distribution.
7. For Normal distribution μ_{2n} =__________.
8. For Normal distribution $\mu_4 = k_4 + 3k_2^2$ is ____.
9. If two independent variates $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ then $X_1 + X_2$ is distributed as ______.
10. If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1+X_2}$ is distributed as______.
11. If two independent variates $X_1 \sim \Lambda(\mu_1, \sigma_1^2)$ and $X_2 \sim \Lambda(\mu_2, \sigma_2^2)$ then $X_1 \cdot X_2$ is distributed as ____.
12. Weibull distribution has application in _________________.
13. If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is _________.
14. Pearson's coefficient of skewness for Chi-square distribution curve is ________.
15. $t -$ distribution curve in respect of tails is always__________.
16. Given a joint Bivariate Normal distribution of X,Y as $BVN(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, the marginal distribution $f_Y(y)$ = _____________.
17. If the variables (X, Y) follow $BVN(1, 2, 4, 9, 0.5)$ distribution, the conditional distribution of $(X/Y = 5)$ has mean ___ and variance______.
18. A measure of linear association of a variable say, X_1 with a number of other variables $X_2, X_3, X_4, \dots, X_k$ is known as ________________.
19. The range of multiple correlation coefficient R is________.
20. Partial correlation coefficient is the simple correlation between ____________variables.

2(a) Write the answer any THREE (Each 2 marks) **(06)**

1. Why characteristic function need?
2. If $u = \frac{x-a}{h}$, a and h being constants then $\phi_u(t) = e^{(-\iota at/h)}\phi_x(t/h)$
3. Define Weibul distribution.
4. Define truncated distribution.

5. Prove that

$$\sigma_{3.12}^2 = \frac{\sigma_3^2(1 - r_{12}^2 - r_{23}^2 - r_{13}^2 + 2r_{12}r_{23}r_{13})}{(1 - r_{12}^2)}$$

6. In trivariate distribution it is found that $r_{12} = 0.77, r_{13} = 0.72$ and $r_{23} = 0.52$. Find (i) $r_{12\,3}$ (ii) $R_{1\,23}$

(b) Write the answer any THREE (Each 3 marks) **(09)**

1. Obtain Probability density function for the characteristic function $\emptyset_X(t) = p\left(1 - qe^{it}\right)^{-1}$
2. Obtain mean and variance of Beta distribution of Second kind.
3. Define Exponential distribution and obtain its Moment Generating Function (MGF). From MGF obtain its mean and variance.
4. Define truncated Poisson distribution and also obtain its mean and variance.
5. Usual notation of multiple correlation and multiple regression, prove that

$$b_{12} = \frac{b_{12.3} + b_{13.2}b_{32\,1}}{1 - b_{13\,2}b_{31\,2}}$$

6. Usual notation of multiple correlation and multiple regression, prove that

$$\sigma_{1\,23}^2 = \sigma_1^2(1 - r_{12}^2)(1 - r_{13\,2}^2)$$

(c) Write the answer any TWO (Each 5 marks) **(10)**

1. State and Prove that Chebchev's inequality.
2. Drive F-distribution.
3. If x and y are independent χ^2 variates with n_1 and n_2 degree of freedom respectively then obtain distribution of $\frac{x}{x+y}$ and $x + y$.
4. Obtain conditional distribution of y when x is given for Bi-variate distribution.
5. Usual notation of multiple correlation and multiple regression, prove that

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{23}^2}$$

3(a) Write the answer any THREE (Each 2 marks) **(06)**

1. Define Beta-I and Beta-II distribution.
2. Define Log Normal distribution when $y = log_e(x - a)$
3. Obtain characteristic function of Poisson distribution with parameter λ
4. Obtain relation between t and F distribution.
5. Usual notion of multiple correlation and multiple regression, prove that

$$R_{1.23}^2 = b_{12.3}r_{12}\frac{\sigma_2}{\sigma_1} + b_{13\,2}r_{13}\frac{\sigma_3}{\sigma_1}$$

6. Prove that $b_{12.3} = \frac{b_{12} - b_{13}b_{23}}{1 - b_{13}b_{23}}$

(b) Write the answer any THREE (Each 3 marks) **(09)**

1. Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r}\emptyset_X(t)\right]_{t=0}$
2. Obtain Moment Generating Function (MGF) of Normal distribution.
3. Obtain mean and variance of Uniform Distribution.
4. Obtain Harmonic mean of $X \sim \gamma(\alpha, p)$.
5. Usual notation of multiple correlation and multiple regression, prove that

$$r_{xy} + r_{yz} + r_{xz} \geq -\frac{3}{2}$$

6. Usual notation of multiple correlation and multiple regression, prove that If $r_{12} = r_{23} = r_{31} = r$ then $R_{1.23} = R_{2.31} = R_{3.12} = \frac{\sqrt{2}r}{\sqrt{1+r}}$

(c) Write the answer any TWO (Each 5 marks) **(10)**

1. Obtain Moment Generating Function (MGF) and Cumulate Generating Function (CGF) of Gamma distribution with parameters α and p. Also show that $\mu_4 = k_4 + 3k_2^2$.
2. Drive Normal distribution.
3. Drive t-distribution.
4. Drive χ^2distribution and show that $3\beta_1 - 2\beta_2 - 6 = 0$.
5. Usual notation of multiple correlation and multiple regression, prove that

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$